Introduction to Logic (MA & Guests)
Q & A for Midterm, Practice midterm 2018

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Question 1: Translation into propositional logic (10 points)

Translate the following sentences into *propositional logic*. Atomic sentences are represented by uppercase letters. Do not forget to provide the translation key — one key for the whole exercise. Represent as much logical structure as possible.

1. The child likes Lego only if she does not like Playmobil.
2. If the child likes wooden toys, then she likes neither Lego nor Playmobil.

Question 1: Translation into propositional logic (10 points)

Translate the following sentences into *propositional logic*. Atomic sentences are represented by uppercase letters. Do not forget to provide the translation key — one key for the whole exercise. Represent as much logical structure as possible.

1. The child likes Lego only if she does not like Playmobil.
2. If the child likes wooden toys, then she likes neither Lego nor Playmobil.

Answer

Translation key:

- L: The child likes Lego.
- P: The child likes Playmobil.
- W: The child likes wooden toys.

Formalizations:

1. $L \to \neg P$
2. $W \to (\neg L \land \neg P)$

Question 2: Translation into first-order logic (10 points)

Translate the following sentences to *first-order logic*. Do not forget to provide the translation key — one key for the whole exercise. Represent as much logical structure as possible.

1. The Linnaeusborg is taller than the Bernoulliborg but the Bernoulliborg is easier to navigate than both the Linneausborg and Nijenborgh.
2. The Linneausborg is easier to navigate than Nijenborgh if and only if neither Nijenborgh nor the Bernoulliborg is red.

Question 2: Translation into first-order logic (10 points)

Translate the following sentences to *first-order logic*. Do not forget to provide the translation key — one key for the whole exercise. Represent as much logical structure as possible.

1. The Linnaeusborg is taller than the Bernoulliborg but the Bernoulliborg is easier to navigate than both the Linneausborg and Nijenborgh.
2. The Linneausborg is easier to navigate than Nijenborgh if and only if neither Nijenborgh nor the Bernoulliborg is red.

Answer Translation key:

- $\text{Taller}(x, y)$: x is taller than y
- $\text{Easier}(x, y)$: x is easier to navigate than y
- $\text{Red}(x)$: x is red
- l: the Linnaeusborg
- b: the Bernoulliborg
- n: Nijenborgh

Formalizations:

1. $\text{Taller}(l, b) \land \text{Easier}(b, l) \land \text{Easier}(b, n)$
2. $\text{Easier}(l, n) \leftrightarrow (\neg\text{Red}(n) \land \neg\text{Red}(b))$

Question 3: Formal proofs (30 points)

Give formal proofs of the following inferences. Do not forget to provide justifications in the correct order. You may only use the Introduction and Elimination rules and the Reiteration rule.

a

$$\begin{array}{|l} a = b \land c = b \\ \neg P(a) \\ \hline \neg P(c) \end{array}$$

b

$$\begin{array}{|l} \\ \hline (\neg Q \to Q) \to \neg\neg Q \end{array}$$

c

$$\begin{array}{|l} A \lor \neg C \\ C \lor \neg B \\ \hline B \to A \end{array}$$

Answer to Question 3 a

To prove:

$$\begin{array}{|l} a = b \wedge c = b \\ \neg P(a) \\ \hline \neg P(c) \end{array}$$

First, set up the main structure:

$$\begin{array}{|l} 1.\ \ a = b \wedge c = b \\ 2.\ \ \neg P(a) \\ \hline \\ \\ \\ \\ \\ \neg P(c) \end{array}$$

Answer to Question 3 a, continued

To prove:

$$\begin{array}{|l} a = b \wedge c = b \\ \neg P(a) \\ \hline \neg P(c) \end{array}$$

Then, fill in the intermediate steps:

1.	$a = b \wedge c = b$	
2.	$\neg P(a)$	
3.	$a = b$	$\wedge$Elim: 1
4.	$c = b$	$\wedge$Elim: 1
5.	$\neg P(b)$	$=$Elim: 2,3
6.	$c = c$	$=$Intro
7.	$b = c$	$=$Elim: 6,4
8.	$\neg P(c)$	$=$Elim: 5,7

Answer to Question 3 a, continued

Also correct:

1.	$a = b \land c = b$	
2.	$\neg P(a)$	
3.	$a = b$	$\land$Elim: 1
4.	$c = b$	$\land$Elim: 1
5.	$c = c$	$=$Intro
6.	$b = c$	$=$Elim: 5,4
7.	$a = c$	$=$Elim: 3, 6
8.	$\neg P(c)$	$=$Elim: 2, 7

Answer to Question 3 b

To prove: $\vdash (\neg Q \to Q) \to \neg\neg Q$

First, set up the main structure:

1. $\neg Q \to Q$

j. $\neg\neg Q$

j+1. $(\neg Q \to Q) \to \neg\neg Q$ $\quad\quad$ $\to$Intro: 1–j

Answer to Question 3 b, Continued

To prove: $\vdash (\neg Q \to Q) \to \neg\neg Q$

Then, fill in the needed subproof:

1. $\neg Q \to Q$	
2. $\neg Q$	
3. Q	$\to$Elim: 1,2
4. $\perp$	$\perp$Intro: 3,2
5. $\neg\neg Q$	$\neg$Intro: 2-4
6. $(\neg Q \to Q) \to \neg\neg Q$	$\to$Intro: 1-5

Answer to Question 3 c

To prove:

$$\begin{array}{|l} A \vee \neg C \\ C \vee \neg B \\ \hline B \rightarrow A \end{array}$$

First, set up the main structure:

1. $A \vee \neg C$	
2. $C \vee \neg B$	
3. B	
j. A	
j+1. $B \rightarrow A$	$\rightarrow$Intro: 3–j

Answer 3c

1. $A \vee \neg C$	
2. $C \vee \neg B$	
3. B	
4. C	
5. C	Reit: 4
6. $\neg B$	
7. $\bot$	$\bot$Intro: 3,6
8. C	$\bot$Elim: 7
9. C	$\vee$Elim: 2, 4-5, 6-8
10. A	
11. A	Reit: 10
12. $\neg C$	
13. $\bot$	$\bot$Intro: 9,12
14. A	$\bot$Elim: 13
15. A	$\vee$Elim:1, 10-11, 12-14
16. $B \rightarrow A$	$\rightarrow$Intro: 3-15

Question 4: Truth tables (15 points)

Use *truth tables* to answer the next questions. Make the full truth tables, and do not forget to draw explicit conclusions from the truth tables in order to explain your answers. Order the rows in the truth tables as follows:

P	Q	R	...
T	T	T	...
T	T	F	...
T	F	T	...
T	F	F	...
F	T	T	...
F	T	F	...
F	F	T	...
F	F	F	...

a = b	b = c	c = a	...
T	T	T	...
T	T	F	...
T	F	T	...
T	F	F	...
F	T	T	...
F	T	F	...
F	F	T	...
F	F	F	...

a) Is $((P \leftrightarrow Q) \leftrightarrow \neg R) \leftrightarrow ((P \leftrightarrow Q) \wedge (Q \leftrightarrow \neg R))$ a *tautology*?

b) Is the sentence $(a = b \wedge \neg(b = c)) \rightarrow \neg(c = a)$ a *logical truth*? Explain your answer and indicate the spurious rows in the truth table.

Answer to Question 4a

P	Q	R	$((P \leftrightarrow Q)$	$\leftrightarrow$	$\neg R)$	$\leftrightarrow$	$((P \leftrightarrow Q)$	$\wedge$	$(Q \leftrightarrow$	$\neg R))$
T	T	T	T	F	F	**T**	T	F	F	F
T	T	F	T	T	T	**T**	T	T	T	T
T	F	T	F	T	F	**F**	F	F	T	F
T	F	F	F	F	T	**T**	F	F	F	T
F	T	T	F	T	F	**F**	F	F	F	F
F	T	F	F	F	T	**T**	F	F	T	T
F	F	T	T	F	F	**F**	T	T	T	F
F	F	F	T	T	T	**F**	T	F	F	T
			(*i*)	(*ii*)	(*i*)	(*iv*)	(*i*)	(*iii*)	(*ii*)	(*i*)

The sentence $((P \leftrightarrow Q) \leftrightarrow \neg R) \leftrightarrow ((P \leftrightarrow Q) \wedge (Q \leftrightarrow \neg R))$ *is not* a tautology, because there is at least one "F" under the main connective in column (iv), namely in the third, fifth, seventh, and eight rows. (Naming at least one of these four counter-examples is sufficient.)

Answer to Question 4b

$a = b$	$b = c$	$c = a$	$(a = b \wedge \neg(b = c)) \to \neg(c = a)$	
T	T	T	F F **T** F	
T	T	F	F F **T** T	spurious row!
T	F	T	T T **F** F	spurious row!
T	F	F	T T **T** T	
F	T	T	F F **T** F	spurious row!
F	T	F	F F **T** T	
F	F	T	F T **T** F	
F	F	F	F T **T** T	
			$(ii)(i)$ $(iii)(i)$	

The sentence $(a = b \wedge \neg(b = c)) \to \neg(c = a)$ *is* a logical truth because we have a "T" in the column under the main connective in all non-spurious rows.

Question 5: Normal forms of propositional logic (15 points)

a Provide a negation normal form (NNF) of this sentence:
$(\neg Q \vee R) \rightarrow \neg(\neg P \leftrightarrow Q)$

b Provide a disjunctive normal form (DNF) of this sentence:
$\neg((Q \rightarrow R) \wedge (R \rightarrow P)) \vee \neg(P \rightarrow Q)$

Indicate the intermediate steps. You do not have to provide justifications for the steps.

Answer to Question 5a

Goal: to rewrite the sentence into a logically equivalent sentence that is in negation normal form (NNF).

$$
\begin{array}{lll}
& (\neg Q \vee R) \rightarrow \neg(\neg P \leftrightarrow Q) & \\
\Leftrightarrow & \neg(\neg Q \vee R) \vee \neg(\neg P \leftrightarrow Q) & \text{def} \rightarrow \\
\Leftrightarrow & \neg(\neg Q \vee R) \vee \neg((\neg P \rightarrow Q) \wedge (Q \rightarrow \neg P)) & \text{def} \leftrightarrow \\
\Leftrightarrow & (\neg\neg Q \wedge \neg R) \vee \neg((\neg P \rightarrow Q) \wedge (Q \rightarrow \neg P)) & \text{De Morgan} \\
\Leftrightarrow & (Q \wedge \neg R) \vee \neg((\neg P \rightarrow Q) \wedge (Q \rightarrow \neg P)) & \text{double } \neg \\
\Leftrightarrow & (Q \wedge \neg R) \vee (\neg(\neg P \rightarrow Q) \vee \neg(Q \rightarrow \neg P)) & \text{De Morgan} \\
\Leftrightarrow & (Q \wedge \neg R) \vee (\neg(\neg\neg P \vee Q) \vee \neg(Q \rightarrow \neg P)) & \text{def} \rightarrow \\
\Leftrightarrow & (Q \wedge \neg R) \vee (\neg(\neg\neg P \vee Q) \vee \neg(\neg Q \vee \neg P)) & \text{def} \rightarrow \\
\Leftrightarrow & (Q \wedge \neg R) \vee (\neg(P \vee Q) \vee \neg(\neg Q \vee \neg P)) & \text{double } \neg \\
\Leftrightarrow & (Q \wedge \neg R) \vee (\neg P \wedge \neg Q) \vee \neg(\neg Q \vee \neg P)) & \text{De Morgan, assoc} \\
\Leftrightarrow & (Q \wedge \neg R) \vee (\neg P \wedge \neg Q) \vee (\neg\neg Q \wedge \neg\neg P) & \text{De Morgan} \\
\Leftrightarrow & (Q \wedge \neg R) \vee (\neg P \wedge \neg Q) \vee (Q \wedge P) & \text{double } \neg
\end{array}
$$

Answer to Question 5a

Goal: to rewrite the sentence into a logically equivalent sentence that is in negation normal form (NNF).

$$
\begin{array}{lll}
& (\neg Q \vee R) \rightarrow \neg(\neg P \leftrightarrow Q) & \\
\Leftrightarrow & \neg(\neg Q \vee R) \vee \neg(\neg P \leftrightarrow Q) & \text{def} \rightarrow \\
\Leftrightarrow & \neg(\neg Q \vee R) \vee \neg((\neg P \rightarrow Q) \wedge (Q \rightarrow \neg P)) & \text{def} \leftrightarrow \\
\Leftrightarrow & (\neg\neg Q \wedge \neg R) \vee \neg((\neg P \rightarrow Q) \wedge (Q \rightarrow \neg P)) & \text{De Morgan} \\
\Leftrightarrow & (Q \wedge \neg R) \vee \neg((\neg P \rightarrow Q) \wedge (Q \rightarrow \neg P)) & \text{double } \neg \\
\Leftrightarrow & (Q \wedge \neg R) \vee (\neg(\neg P \rightarrow Q) \vee \neg(Q \rightarrow \neg P)) & \text{De Morgan} \\
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\Leftrightarrow & (Q \wedge \neg R) \vee (\neg P \wedge \neg Q) \vee \neg(\neg Q \vee \neg P)) & \text{De Morgan, assoc} \\
\Leftrightarrow & (Q \wedge \neg R) \vee (\neg P \wedge \neg Q) \vee (\neg\neg Q \wedge \neg\neg P) & \text{De Morgan} \\
\Leftrightarrow & (Q \wedge \neg R) \vee (\neg P \wedge \neg Q) \vee (Q \wedge P) & \text{double } \neg
\end{array}
$$

Several steps can be skipped by using the equivalence
$\neg(A \rightarrow C) \Leftrightarrow (A \wedge \neg C)$

Answer to Question 5b

Goal: to rewrite the sentence into a logically equivalent sentence that is in disjunctive normal form (DNF).

$$\neg((Q \rightarrow R) \land (R \rightarrow P)) \lor \neg(P \rightarrow Q)$$

$$\begin{array}{lll}
\Leftrightarrow & \neg(Q \rightarrow R) \lor \neg(R \rightarrow P) \lor \neg(P \rightarrow Q) & \text{De Morgan, assoc} \\
\Leftrightarrow & \neg(Q \rightarrow R) \lor \neg(R \rightarrow P) \lor (P \land \neg Q) & \text{def} \rightarrow \\
\Leftrightarrow & \neg(Q \rightarrow R) \lor (R \land \neg P) \lor (P \land \neg Q) & \text{def} \rightarrow \\
\Leftrightarrow & (Q \land \neg R) \lor (R \land \neg P) \lor (P \land \neg Q) & \text{def} \rightarrow
\end{array}$$

Question 6: Set theory (10 points)

Consider the following three sets:

$$A = \{\emptyset, a\}, \quad B = \{\emptyset, \{a\}\} \quad \text{and } C = \{\emptyset, \{a\}, \{\emptyset, \{a\}\}\}.$$

For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

1. $a \in A$

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For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

1. $a \in A$ Yes

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For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

1. $a \in A$ Yes
2. $A \subseteq B$

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For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

1. $a \in A$ Yes
2. $A \subseteq B$ No

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For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

1. $a \in A$ Yes
2. $A \subseteq B$ No
3. $A \cap B = \emptyset$

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2. $A \subseteq B$ No
3. $A \cap B = \emptyset$ No
4. $B \cap C = B$

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1. $a \in A$ Yes
2. $A \subseteq B$ No
3. $A \cap B = \emptyset$ No
4. $B \cap C = B$ Yes
5. $A \subseteq C$

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Consider the following three sets:

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3. $A \cap B = \emptyset$ No
4. $B \cap C = B$ Yes
5. $A \subseteq C$ No

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$$A = \{\emptyset, a\}, \;\; B = \{\emptyset, \{a\}\} \;\text{ and }\; C = \{\emptyset, \{a\}, \{\emptyset, \{a\}\}\}.$$

For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

1. $a \in A$ Yes
2. $A \subseteq B$ No
3. $A \cap B = \emptyset$ No
4. $B \cap C = B$ Yes
5. $A \subseteq C$ No
6. $C = B \cup \{B\}$

Question 6: Set theory (10 points)

Consider the following three sets:

$$A = \{\emptyset, a\}, \quad B = \{\emptyset, \{a\}\} \quad \text{and } C = \{\emptyset, \{a\}, \{\emptyset, \{a\}\}\}.$$

For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

1. $a \in A$ Yes
2. $A \subseteq B$ No
3. $A \cap B = \emptyset$ No
4. $B \cap C = B$ Yes
5. $A \subseteq C$ No
6. $C = B \cup \{B\}$ Yes

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Consider the following three sets:

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For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

1. $a \in A$ Yes
2. $A \subseteq B$ No
3. $A \cap B = \emptyset$ No
4. $B \cap C = B$ Yes
5. $A \subseteq C$ No
6. $C = B \cup \{B\}$ Yes
7. $\emptyset \in A \cap B \cap C$

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Consider the following three sets:

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For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

1. $a \in A$ Yes
2. $A \subseteq B$ No
3. $A \cap B = \emptyset$ No
4. $B \cap C = B$ Yes
5. $A \subseteq C$ No
6. $C = B \cup \{B\}$ Yes
7. $\emptyset \in A \cap B \cap C$ Yes

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For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

1. $a \in A$ Yes
2. $A \subseteq B$ No
3. $A \cap B = \emptyset$ No
4. $B \cap C = B$ Yes
5. $A \subseteq C$ No
6. $C = B \cup \{B\}$ Yes
7. $\emptyset \in A \cap B \cap C$ Yes
8. $(B \backslash A) = \{a, \{a\}\}$

Question 6: Set theory (10 points)

Consider the following three sets:

$$A = \{\emptyset, a\}, \;\; B = \{\emptyset, \{a\}\} \;\text{ and }\; C = \{\emptyset, \{a\}, \{\emptyset, \{a\}\}\}.$$

For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

1. $a \in A$ Yes
2. $A \subseteq B$ No
3. $A \cap B = \emptyset$ No
4. $B \cap C = B$ Yes
5. $A \subseteq C$ No
6. $C = B \cup \{B\}$ Yes
7. $\emptyset \in A \cap B \cap C$ Yes
8. $(B \backslash A) = \{a, \{a\}\}$ No

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$$A = \{\emptyset, a\}, \;\; B = \{\emptyset, \{a\}\} \;\text{ and }\; C = \{\emptyset, \{a\}, \{\emptyset, \{a\}\}\}.$$

For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

1. $a \in A$ Yes
2. $A \subseteq B$ No
3. $A \cap B = \emptyset$ No
4. $B \cap C = B$ Yes
5. $A \subseteq C$ No
6. $C = B \cup \{B\}$ Yes
7. $\emptyset \in A \cap B \cap C$ Yes
8. $(B \backslash A) = \{a, \{a\}\}$ No
9. $C \subseteq A \cup B$

Question 6: Set theory (10 points)

Consider the following three sets:

$$A = \{\emptyset, a\}, \quad B = \{\emptyset, \{a\}\} \text{ and } C = \{\emptyset, \{a\}, \{\emptyset, \{a\}\}\}.$$

For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

1. $a \in A$ Yes
2. $A \subseteq B$ No
3. $A \cap B = \emptyset$ No
4. $B \cap C = B$ Yes
5. $A \subseteq C$ No
6. $C = B \cup \{B\}$ Yes
7. $\emptyset \in A \cap B \cap C$ Yes
8. $(B \setminus A) = \{a, \{a\}\}$ No
9. $C \subseteq A \cup B$ No

Question 6: Set theory (10 points)

Consider the following three sets:

$$A = \{\emptyset, a\}, \;\; B = \{\emptyset, \{a\}\} \;\text{ and } C = \{\emptyset, \{a\}, \{\emptyset, \{a\}\}\}.$$

For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

1. $a \in A$ Yes
2. $A \subseteq B$ No
3. $A \cap B = \emptyset$ No
4. $B \cap C = B$ Yes
5. $A \subseteq C$ No
6. $C = B \cup \{B\}$ Yes
7. $\emptyset \in A \cap B \cap C$ Yes
8. $(B \backslash A) = \{a, \{a\}\}$ No
9. $C \subseteq A \cup B$ No
10. $B \in C \backslash A$

Question 6: Set theory (10 points)

Consider the following three sets:

$$A = \{\emptyset, a\}, \;\; B = \{\emptyset, \{a\}\} \;\text{ and }\; C = \{\emptyset, \{a\}, \{\emptyset, \{a\}\}\}.$$

For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

1. $a \in A$ Yes
2. $A \subseteq B$ No
3. $A \cap B = \emptyset$ No
4. $B \cap C = B$ Yes
5. $A \subseteq C$ No
6. $C = B \cup \{B\}$ Yes
7. $\emptyset \in A \cap B \cap C$ Yes
8. $(B \setminus A) = \{a, \{a\}\}$ No
9. $C \subseteq A \cup B$ No
10. $B \in C \setminus A$ Yes

Question 7: Bonus question (10 points)

Give a formal proof of the following inference:

$$\begin{array}{|l} P \leftrightarrow Q \\ \neg(Q \leftrightarrow R) \\ \hline P \leftrightarrow \neg R \end{array}$$

Do not forget to provide justifications in the correct order. You may only use the Introduction and Elimination rules and the Reiteration rule.

Answer bonus

7

1.	$P \leftrightarrow Q$	
2	$\neg(Q \leftrightarrow R)$	
3	P	
4	Q	$\leftrightarrow$ Elim: 1, 3
5	R	
6	Q	
7	R	Reit: 5
8	R	
9	Q	Reit: 4
10.	$Q \leftrightarrow R$	$\leftrightarrow$ Intro: 6-7, 8-9
11.	$\perp$	$\perp$ Intro: 10, 2
12.	$\neg R$	$\neg$ Intro: 5-11
13	$\neg R$	
14	$\neg Q$	
15	Q	
16	$\perp$	$\perp$ Intro: 15, 14
17	R	$\perp$ Elim: 16
18	R	
19	$\perp$	$\perp$ Intro: 18, 13
20	Q	$\perp$ Elim: 19
21	$Q \leftrightarrow R$	$\leftrightarrow$ Intro: 15-17, 18-20
22	$\perp$	$\perp$ Intro: 21, 2
23	$\neg\neg Q$	$\neg$ Intro: 14-22
24.	Q	$\neg$ Elim: 23
25	P	$\leftrightarrow$ Elim: 1, 24
26	$P \leftrightarrow \neg R$	$\leftrightarrow$ Intro: 3-12, 13-25